DIAGRAMS AS TO ADMITTANCE OF LIGHT TO BUILDINGS.

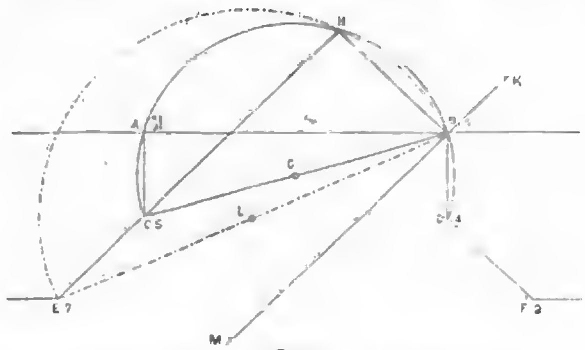

Fig. 1.

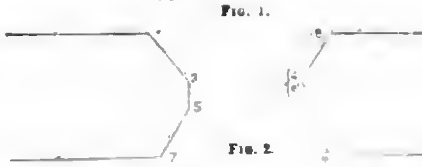

Fig. 2.

Fig. 3

ON THE ADMISSION OF DAYLIGHT INTO BUILDINGS.*

The following demonstrations will serve to make the subject more intelligible:—

Let ABEF (Fig. 1) be a section through a rectangular opening for light exposed to a hemisphere of sky, the section being at right angles with two of the sides of the rectangle. Let the sides AC, BD, of the opening be square with the face, and let CE, DF, be splayed off at equal angles one to the other. Let us consider first only the plane of the section. Join BC and BE, and bisect them in G and L respectively. With centre G rad. GC describe circle CHB, passing through A (because BAC is a right angle). With centre L rad. LE describe circle EHB. Produce EC cutting circle CHB in H. Join HB. Then H is a point in circle EHB also, because EHB is a right angle. Through B draw MK parallel to EH. Now BE is the line of one of the two extreme oblique rays which can pass through the opening (AF being the other). Also from every point in that part of the sky which is intercepted between EB and FA produced, triangles of rays will pass through the opening limited by its sides. These rays, if systematized according to their parallel directions will form parallelograms parallel to every line between BE and AF. Taking these parallelograms in order, from the line of the extreme ray BE to the line of the other extreme ray AF, their widths on BE is O; their widths then continually increase up to the perpendicular to AB, and then continually diminish till on AF they are again O.

A very approximate proportional value of these parallelograms may be obtained by taking the widths at certain small equal angular intervals as they pass from BE to AF. These small intervals will subtend equal angles so long as the centre lines of the successive parallelograms pass through the centres of the circles on which the degrees are measured, and let us take *two* degrees as the equal intervals. Take any such parallelogram HEKM—then ∵ CHB is a right angle, ∴ BH (or chord of arc BH) measures its width. So the chords of the arcs from B to BH (rad. LB), taken at two degrees intervals, measure all the parallelograms at such intervals as far as the parallel of EH. Thus far B and E have been the points which limit the widths of the parallelograms, but on passing the parallel of EH, B and C become the limiting points. At this parallel BH is the chord of both circles BHE and BHC. Also the chords from arc BH to arc BHA taken at such 2° intervals will measure all the remaining parallelograms at similar intervals, as far as the perpendicular to AB. ∴ the sum of the chords of arcs from B to arc BH (rad. LB) added to the sum of the chords from arc BH to arc BHA (rad. GB), will measure proportionally all the parallelograms passing at such intervals through the opening from BE to the said perpendicular to AB. Also the same will measure the parallelograms passing through the opening from AF to the said perpendicular to AB. ∴ twice the said sums of chords will measure all the parallelograms passing at such intervals through the opening on the plane of the said section. But the chord of an arc = 2 × sine of ½ the arc. ∴ (taking now degrees as the intervals instead of 2 degrees) 2 × sum of sines from B to ½ arc BH × rad. LB + 2 × sum of sines from ½ arc BH to ½ arc BHA × rad. GB measure the same.

But ½ arc BH rad. LB is represented by ∠ BEH (which = ½ ∠ BLH.)
And ½ arc BH rad. GB is represented by ∠ BCH (which = ½ ∠ BGH.)
And ½ arc BHA rad. GB is represented by ∠ BCA (which = ½ ∠ BGA.)

Now, putting the diameter* BE for twice the rad. LB and the diameter* BC for twice the rad. GB, we have: BE × sum of sines from 0° to ∠ BEC + BC × sum of sines from BCE (supplement of BCH) to BCA measure the same. A similar formula will apply to the section perpendicular to the other two sides of the rectangular opening, and the results of the two formulæ multiplied together will give *a number proportionate to the quantity of light passing through the opening.*

The following general theorem may be deduced in like manner. Let Fig. 2 represent a section perpendicular to two sides of a rectangular opening, the sides being irregular. Figure the two points which are in the line of one of the extreme rays 1, 2. Then considering the successive parallelograms which would pass through the opening, beginning from line 1, 2, these points 1 and 2 will at first limit the width of such parallelograms. The succession continuing, another point will, perhaps, become one of the limiting points instead of either 1 or 2. Let this point be instead of 1, and figure it 3; say the next limiting point is instead of point 2, and figure it 4; the next is (say) instead of point 3, and figure it 5; now suppose the next limiting point is instead of 5 (and therefore on the same side of the opening as 5): in this case recur to point 4, and assign it a second figure 6, then figure the limiting point instead of 5, 7; let the next be instead of point $\frac{4}{6}$ and figure it 8, and so on. The even numbers will thus be on the one side, and the odd numbers on the other side of the opening. This being done the table which follows will give the proper result:—

Diagonals.		*Sum of Sines.*	
1—2	×	from 0 to ∠ 213	
2—3	×	„ ∠ 132 to ∠ 324	
3—4	×	„ ∠ 243 to 90 + 90 to ∠ 435	
{4—5	×	„ ∠ 354 to ∠ 657	× ½ =
{5—6}			
6—7	×	„ ∠ 576 to ∠ 768	
7—8	×	„ ∠ 687 to 0°	
&c.			

The following are the rules for constructing the table. (1.) Put diagonals in order 1. 2, 2. 3, &c. as far as the figures on the diagram extend. (2.) If the same diagonal is represented on two lines, bracket them as $\begin{Bmatrix} 4\,.\,5 \\ 5\,.\,6 \end{Bmatrix}$ (3.) Put (or suppose) zigzag lines from 2 to 1, 1 to 3, 3 to 2, &c. as shown. This forms the first column of *diagonals*. The second column shows the angles between which the *sums of the sines* are to be taken at successive intervals of degrees. (4.) Look to the column of diagonals and take the angles in the zigzag order, as shown 213, ∠ 132, 324, &c. The first line will be from 0° to ∠ 213, the second ∠ 132 to ∠ 324, &c.; and the last line will be from the last three figures (as above, 687) to 0°. (5.) If two obtuse angles come in the same line (as in the third line above), put down from first angle to 90° added to 90° to the second angle. (6.) The third column is the multiple ½. The use of the table is as follows:—(1.) Measure the diagonals 1. 2, 2. 3, &c. and put down the measures in order in feet and decimals. (2.) Measure the angles 213, 132, &c. and put down the number of degrees in order. (3.) Find from the table *below* the sums of sines (taking the supplements where obtuse) and note them in their place. (4.) The result is found as follows: Diag. 1. 2 × sum of sines from 0° to ∠ 213 + diag. 2. 3 × sum of sines from ∠ 132 to ∠ 324, and so on. The sum of the quotients to be multiplied into the third column (½).

So long a formula will seldom be required in practice. Where the sides of the opening are similar it will be sufficient to go only up to 90° from the line of the extreme ray. Thus, in Fig. 1, which is figured according to the rule, the formula is—

Diagonals.		*Sum of Sines.*	
{1 — 2}	×	from 0° to 321	
{2 — 3}			=
3 — 4	×	„ 243 to 435	
4 — 5	×	„ 354 to 546	

The third line is nil, because 354 and 546 are both right angles. The third column (½) of the formula is cancelled, because we have

* See p. 364, ante.

* These are hereafter called diagonals, as more expressive of lines passing through the angles formed by the sides of the opening.